Cosmological Quantum String Vacua and String-String Duality[†]

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ABSTRACT

Implications of string-string dualities to cosmological string vacua are discussed. Cosmological vacua of classical string theories comprise of disjoint classses mapped one another by scale-factor T-duality. Each classes are, however, afflicted with initial/final cosmological singularities. It is argued that quantum string theories and string-string dualities dramatically resolve these cosmological singularities out so that disjoint classical cosmological vacua are continuously connected in a unified manner. A natural inflationary cosmology follows from this.

Cosmological Quantum String Vacua and String-String Duality

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Implications of string-string dualities to cosmological string vacua are discussed. Cosmological vacua of classical string theories comprise of disjoint classses mapped one another by scale-factor T-duality. Each classes are, however, afflicted with initial/final cosmological singularities. It is argued that quantum string theories and string-string dualities dramatically resolve these cosmological singularities out so that disjoint classical cosmological vacua are continuously connected in a unified manner. A natural inflationary cosmology follows from this.

Tremendous progress in recent years to non-perturbative string theory [1] has now shown compelling evidences that all known perturbative string theories (type-I, heterotic, type II) are equivalent nonperturbatively up to duality transformations [2]. String dualities have been studied so far, however, only for compactifications to Minkowski spacetime. Cosmological string vacua are described by an 'adiabatic motion' of spacetime geometry in the string moduli space, viz. a fibration of three-geometry, string coupling and compactified space over 'cosmological time'. It is then of immediate interest what the string dualities might tell us for string cosmology. In particular we would like to know if the string dualities can offer a resolution to cosmological singularity in a similar spirit to the conifold transition [3]. In this talk, based on a simple two-dimensional compactification, I exemplify that string dualities offer an interesting cosmological quantum string vacua with an inflation but no initial or final singularities [4].

String theories exhibit generically two interesting classical cosmological vacua [5]: (1) Friedmann-Robertson-Walker (FRW)-like vacua of decelerating expansion $\ddot{a}(t) < 0$, $\dot{a}(t) > 0$, (2) super-inflationary vacua of accelerating expansion $\ddot{a}(t) > 0$, $\dot{a}(t) > 0$, related each other under scale factor T-duality accompanied by a time-

reversal. The two vacua are, however, afflicted with initial/final singularities of spacetime geometry. In fact, scale factor T-duality (accompanied by a time-reversal) relates the two vacua, but offers no resolution of the singularities. Because of this, the classical string cosmology is afflicted by a 'graceful exit problem' [6].

Nature of the problem and quantum resolution thereafter can be treated exactly in two-dimensional sting compactifications. A given compactification is specified by spacetime geometry, dilaton and various compactification moduli, and is described by the same action as the dilaton gravity [7] with vanishing central charge deficit (cosmological constant)

$$e^{-1}L_0 = [e^{-2\phi}(R + 4(\nabla\phi)^2) - \frac{1}{2}(\nabla\vec{f})^2] \qquad (1)$$

where ϕ and $\vec{f}$ refer to dilaton and N-component Ramond-Ramond scalar field. In conformal gauge $ds^2 = -e^{2\rho}dx_+dx_-$; $x_\pm = t \pm x$, $\partial_\pm = \frac{1}{2}(\cdot \pm \prime)$, the Lagrangian is expressed in terms of field variables $\Phi = e^{-2\phi}$, $\Sigma = 2\kappa(\phi - \rho)$ as

$$L_0 = \frac{1}{2\kappa}(\partial_+\Phi\partial_-\Sigma + \partial_-\Phi\partial_+\Sigma) + \frac{1}{2}\partial_+\vec{f}\cdot\partial_-\vec{f} \quad (2)$$

supplemented with constraint equations

$$T_{\pm\pm} = \frac{1}{2}(\partial_\pm\vec{f})^2 + \partial_\pm^2\Phi + \frac{1}{\kappa}\partial_\pm\Phi\partial_\pm\Sigma = 0. \qquad (3)$$

A parameter κ is introduced to keep track of perturbation expansions.

In the classical limit $\kappa \to 0$, the theory exhibit two useful symmetries $\Phi \to \Phi + \epsilon_\Sigma$, $\Sigma \to \Sigma$ and

$\Sigma \to \Sigma + \epsilon_\Phi, \Phi \to \Phi$, thus exactly soluble. The most general cosmological solution to equations of motion $\ddot{\Phi} = \ddot{\Sigma} = 0$, $\vec{f} = \text{constant}$ are

$$-\frac{1}{2\kappa}\Sigma = Q_\Sigma t + A, \quad \Phi = Q_\Phi t + B \tag{4}$$

subject to Eq.(3) $T_{\pm\pm} = 2Q_\Phi Q_\Sigma = 0$.

For the vacua with $Q_\Phi \neq 0$: $\rho = \phi + \log 2\tilde{M}$, $e^{-2\phi} = -8\tilde{M}t$, the spacetime is given by

$$(ds)^2 = -[d\tau^2 - (\frac{\tilde{M}}{-\tau})^2 dx^2]; \quad -\infty < \tau \leq 0 \tag{5}$$

with cosmic time $\tau := -(-2\tilde{M}t)^{1/2}$. The co-moving scale factor $a(\tau) \propto 1/(-\tau)$ shows that this vacua describes a super-inflationary evolution. Dilaton grows large as $\phi = -\log(-2\tau)$.

For the other vacua with $Q_\Sigma \neq 0$: $\rho = \phi + Mt$, $e^{-2\phi} = M^{-2}$, the spacetime is given by

$$(ds)^2 = -[d\tau^2 - (M\tau)^2 dx^2]; \quad 0 \leq \tau < \infty \tag{6}$$

with $\tau := \exp Mt$, $0 \leq \tau < \infty$. This linearly expanding unverse with a constant dilaton is a two-dimensional counterpart of radiation-dominated FRW-type vacua.

Both vacua are afflicted by divergent curvature $R = (2/|\tau|)^2 \to \infty$ at $\tau = 0$, viz., cosmological initial or final singularities. While the two vacua are related each other by scale factor T-duality and time-reversal: $-\tilde{M}/\tau \to M\tau$, patching them together using the T-duality does not remove the singularity since both curvature and string coupling remain singular and discontinuous across the junction $\tau = 0$. This difficulty persists even after higher-derivate α' corrections are included (also in $D = 4$); it is the the aformentioned *graceful exit problem* [6].

There is a good reason, however, to expect that the situation might be changed for *quantum* string vacua. Near the T-duality junction, $\tau \approx 0$, the string coupling diverges, hence, back reaction will become important [8]. At one-loop, quantum correction is provided by conformal anomaly of N-component matter, reparametrization ghosts, dilaton and conformal modes. For $N > 5$ supersymmetric compactifications (corresponding to $D = 4, N > 2$ ones), the quantum correction is saturated at one-loop, hence, the above consideration yields an exact quantum theory. After adding a local counterterm to retain the classical symmetries for exact solvability [9], the quantum effective Lagrangian is given by

$$L_{\text{eff}} = L_0 - \frac{\kappa}{2}\big[R\frac{1}{\square}R + 2\phi R\big] \tag{7}$$

where $\kappa = (N - 24)/24$. In terms of quantum-corrected fields $\Sigma = 2\kappa(\phi - \rho)$; $\Phi = e^{-2\phi} + \kappa\rho$ $L_{|rmeff}$ is exactly the same as classical L_o, but the constraint equation is corrected to

$$T_{\pm\pm} - \kappa\big(\frac{1}{2}\partial_\pm\Sigma + t_\pm(x^\pm)\big) = 0. \tag{8}$$

The first integrals $t_\pm(x^\pm)$ coming from nonlocality of conformal anomaly are fixed $t_\pm = 0$ to yield the Minkowski vacuum for a constant dilaton.

It turns out that physically acceptable cosmological *quantum* vacua result in only for negative conformal anomaly $\kappa < 0$, $N < 24$:

$$\frac{1}{2|\kappa|}\Sigma = Q_\Sigma t + A; \quad \Phi = Q_\Phi t + B \tag{9}$$

subject to Eq.(8) $0 = \kappa t_\pm = Q_\Phi Q_\Sigma$.

The first quantum vacua with $Q_\Phi \neq 0$ that exhibited classically a super-inflation is given by $\rho = \phi + \log 2\tilde{M}$; $e^{-2\phi} - |\kappa|\rho = -8\tilde{M}t$. Most importantly quantum correction now extends the cosmic evolution beyond the classical range $-\infty < t \leq 0$ into $-\infty < t < +\infty$ so long as $\kappa < 0$. Noting that ϕ is a monotonic function of t the string coupling $g_{\text{st}} = e^\phi$ may be viewed as a built-in clock. As expected, near $t \approx -\infty$, the vacua exhibit a classical, super-inflation $\dot{a}(\tau) > 0, \ddot{a}(\tau) > 0$. On the other hand, as $t \to +\infty$, $\dot{a}(\tau) > 0$ but $\ddot{a}(\tau) < 0$ and approaches asymptotically to

$$(ds)^2 \to -[d\tau^2 - (\frac{8\tilde{M}}{|\kappa|}\tau)^2 dx^2]; \quad \phi \to \log\tau, \tag{10}$$

where $\tau \approx (|\kappa|/8\tilde{M}) \exp(8\tilde{M}t/|\kappa|)$. The initially super-inflated universe has now decelerated into FRW-like expansion with a linear dilaton. More insight into the interpolating behavior of the quantum vacua can be gained from the scalar curvature as a function of string coupling g_{st}: $R = 16g_{\text{st}}^2/(1 + |\kappa|g_{\text{st}}^2/2)^3$. The curvature vanishes at past/future infinity $g_{\text{st}} \to 0/\infty$ but reaches a finite maximum at T-duality junction $\tau = 0$. Hence, the classical singularity at $\tau = 0$ is now

completely erased out and the inflation has ended gracefully!

Interestingly, the second quantum vacua with $Q_\Sigma \neq 0 : \rho = \phi + Mt;\ e^{-2\phi} - |\kappa|\rho = M^{-2}$ that corresponded classically to a distinct, FRW-like vacua is now related to the first quantum vacua by S-duality. At $t \to +\infty$, the vacua exhibit expected classical FRW-like evolution

$$(ds)^2 \to -[d\tau^2 - (\frac{\tau}{|\kappa|})^2 dx^2]; \phi \to \text{const.} \qquad (11)$$

At $t \to -\infty$, however, the vacua

$$(ds)^2 \to -[d\tau^2 - \exp(2e^{2M\tau})dx^2]; \phi \to -M\tau, (12)$$

evolves initially as a Minkowski spacetime with a linear dilaton then into super-inflation $\dot{a}(\tau) > 0$, $\ddot{a}(\tau) > 0$, $1/H \approx \exp(-2M\tau) \to 0$. Scalar curvature expressed as a funtion of g_{st} takes a suggestive nonperturbative form associated with particle production $R \propto \exp(-\frac{2}{|\kappa|}\frac{1}{g_{\text{st}}^2})g_{\text{st}}^4/(1 + |\kappa|g_{\text{st}}^2/2)^3$. It vanishes at asymptotic past/future infinities $g_{\text{st}} \to \infty/0$ but approaches a finite positive maximum at T-duality junction $\tau = 0$. We thus conclude that the second quantum vacua is S-dual to the first one and both resolve classical singularities and exit super-inflation epoch gracefully. We again emphasize that, no matter how small g_{st} and $|\kappa|$ were, the quantum cosmological vacua behave dramatically different from the classical cosmological vacua, similar to [3].

One common aspect of both quantum vacua is that the string coupling $g_{\text{st}} = e^\phi$ evolves monotonically: either from weak to strong for the first vacua or from strong to weak coupling for the second. The two *quantum* cosmological vacua are S-dual each other and interpolate between the two classical cosmological vacua related by scale-factor T-duality and time-reversal. This is in accord with the emerging picture of string-string dualities: string coupling evolution from weak to strong or vice versa is equivalent to an interpolation between two weakly coupled phases of dual string pairs [2] such as type-I and heterotic and type-II/M-theory and heterotic strings. Interestingly the scale-factor T-duality junction $\tau = 0$ is precisely where g_{st} becomes order unity, hence, where worldsheet and string loop perturbation expansions in both dual pair string threories

break down. Evidently this is where quantum back reaction is most pronounced and the classical singularities are erased completely.

Preliminary study indicates that higher dimensional compactifications also exhibit similar behavior: singularity-free, cosmological quantum string vacua all connected and unified via S- and T-duality transformations. It is undoubtedly gratifying that string dualities seem to offer solutions to the outstanding problems of classical string cosmology.

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